Physics 131 Some extra problems for those that want them! You're welcome! Dr. AP

Estimations, Dimensions, Units…

1. Estimate the amount of sand at the coastal beaches of the United States. You will have to make assumptions about the typical width and depth of a beach and the amount of coastline in the US which has actual sand (rather than rock).

These sorts of problems demand a strong imagination…

1. If you've never seen sand before, you're likely to make some questionable assumptions, but on homework assignments, you're expected to make up any such deficiencies by a quick search online for some details. Still, stick to no more than 2 digits precision while you work this out, and report your final answer in ONE digit precision…
   The US coastline is 133,312 km long according to Wikipedia. Given ***assumptions*** that 1/2 the coastline has beaches of depth 10 m and width 250 m based purely on my personal experiences at beaches, this makes for a volume of sand 1.3×10^8m x 10 m x 200 m ~ 3×10^{11} m^3. Another assumption is that sand is approximately 1 mm across. Now is it spherical? Hardly, but this is an estimate problem. Sphere of radius 1 mm or cube of side 1 mm gives either $4/3\ \pi$ mm^3 ~ 4 mm^3 or just 1 mm^3, so answers can vary quite a bit from the precision you're used to (but should still be within a factor of 10 or so!).

   Remember, you can use "coastline" and "grain of sand" as kinds of units, too. So number of grains of sand per coastline is approximately (3×10^{11} m^3 /coastline) / (4 mm^3/grain of sand).

   The next stumbling block is that since 1 mm = 10^{-3} m, then 1 mm^3 = 10^{-9}m. So our final answer is (about!)(3 x 10^{11} m^3/coastline) / (4 x $10^{-9}m^3$/grain of sand) = 7×10^{19} grains of sand/coastline. It's not Avogadro's number (by a factor of 10,000) but it's still ridiculously large.

2. Bacteria reproduce by doubling every 15-20 minutes. (See, e.g., this source.) Assume you are suffering from an infection of MRSA (Methicillin-resistant *Staphylococcus aureus*) and take a new antibiotic to beat it. Alas, it turns out that 0.01% of your bacterial load are also resistant to this new antibiotic! After the new antibiotic has killed of 99.99% of the MRSA in your body, how long would it take for it to double back to the same load as before you took the antibiotic? Note this question is asking in terms of *percentages* not actual numbers.

Again, you need a good imagination (blech, germs!) to figure this out.

2. I get about 13-14 "cycles", each of 15-20 minutes - so about 3-4 hours. Yikes. What did you get? Did you justify it?
   Since 0.01% survives, or 1 part in 10^4, we need this bug to double a certain number of cycles to get to 10,000: that is, we need to find out what N would give us $2^N = 10^4$. Either you can just multiply 2 over and over again to figure this out, or you could take the log of both sides: $N \log_{10}(2) = 4$; $N = 4/0.3 = 13.3$ (2^{13} ~ 8k and 2^{14} ~ 16k by the calculator game so you know it's somewhere between 13, 14 cycles.) Since each cycle takes 15-20 minutes, we can get a range from (a low of) 13 cycles x 15 minutes = 195 minutes x 1 hr/60 min = 3.25 hr (3 hr 15 min) up to (a high of) 14 cycles x 20 minutes = 280 minutes x 1 hr/60 min = 4.67 hr (4 hr 40 min). Not long…

3. A classic problem presented to physics students is to calculate the speed of light in furlongs per fortnight. Let's at least pretend to make this more biology related. Fast neurons typically reset every 5 ms or so (neuronal firing rate). Imagine a string of neurons, where each neuron is about 100 μm long. How "fast" can changing information be reasonably sent from one end of this string to the other on the basis of a constantly-changing stimulus? Report your answer in μm/ms and in furlongs/fortnight.

3. Since new information can only go as fast as the neuron can reset, 100 μm/5 ms = 20 μm/ms = 2 cm/s ~ 1 inch/s, pretty slow! 1 furlong= 220 yd ~ 201 m; 1 fortnight = 14 days = 1.2×10^6s. Did you get about 120 furlongs/fortnight?

4. Draw two horizontal "number lines", one above the other. Line up "273" of the top one with "32" on the bottom one and "373" on the top one with "212" on the bottom. Clearly these are crazy units with a weird conversion. Not only is the size of each unit different, the "origin" is arbitrarily "off" by some weird value. At what value would the two lines "agree"? Why? Can you derive or infer a formula which can give you the value of one line's

value based on the other's? What would you "do" to that formula to derive where they are equal through math rather than through lengthy number lines?

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4. Fahrenheit and Kelvin, of course! Don't look up the formula until you've attempted to infer it from these two "parallel" number lines:

Effectively, you're assuming that one of them can be put in terms of the other one linearly: $F = mK + b$ (think *y* = *mx* + *b*) from algebra 1). Start by looking at how they *change* (to get the slopes). The top graph (let's call that *K* with malice aforethought) changes 100 points when the bottom line (*F*) changes 180 points: (373 – 273) vs. (212 – 32). This means that $m = \Delta F/\Delta K = 180/100 = 1.8$; by the way, how do I know $m \neq \Delta K/\Delta F$ based on the formula I'm working with: $F = mK + b$? From units! I need *K* to be multiplied by something with new *F* units *divided* by *K* type units to cancel out the *K* units!
Now that we have *m,* plug in one "matching set" of points for the *F* and *K* line…like *F* = 32 and *K* = 273 to figure out what *b* must be: $b = F - mK = 32 - (1.8)273 = -459.4$. The student is challenged now to derive *K* in terms of *F* by following the same prescription (rather than rearranging the one I figured out for you!).
To find out where they're equal, you would set *F* = *K* in whatever formula you came up with and solve for "it" instead of them. I get that they are equal around 574: $F = 1.8F - 459.4$ means that $F = 459.4/0.8 = 574.25$ but I'd never worry about all that precision, would you? (Especially since the freezing point of water is defined in Kelvin as 273.15 K under STP and pure water, etc., etc.) Amusing since the temperature that Celsius and Fahrenheit are "the same" numerically is around –40.